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ON THE RELEVANCE OR IRRELEVANCE OF PUBLIC FINANCIAL POLICY

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Abstract

On the Relevance or Irrelevance of Public Financial Policy

This paper establishes conditions under which public financial policy has neither real nor inflationary effects; under which it has inflationary effects, but not real effects; and under which it has real effects. An increase in government debt (keeping real expenditures fixed), accompanied by a decrease in lump sum taxes has neither inflationary nor real effects (even in a stochastic environment) provided there are no redistribution effects: the increase in supply of government bonds gives rise to an exactly offsetting increase in demand. An increase in the interest rate paid on government debt will be associated with an increase in the rate of inflation, but there will be no real effects. A change in financial policy which preserves the mean rate of return on bonds has no real effects if individuals are risk neutral and changes in the level of debt are offset by changes in lump sum taxes/ subsidies for the owners of bonds. Except in these special cases, changes in public financial policy will always have real effects.

The second part of the paper establishes that the optimal intertemporal risk redistribution scheme can be implemented through financial policies which entail constant price levels. This result is established in the context of a life cycle model with homogeneous individuals. It is shown, furthermore, that only a single financial instrument is required to implement the optimal policy; additional financial instruments are redundant. This redundancy result does not obtain, however, with heterogeneous populations if there are restrictions on the ability of the government to impose differential lump sum taxes on different groups.

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I. Introduction

There is a long-standing belief that while the money supply affects the price level, "real" variables are determined independently. This proposition is generally referred to as the "classical dichotomy." Variants of this belief in the inefficacy of monetary policy, its inability to effect anything real, have regained strength with the emergence of the new classical economics. This belief, however, is far from universal, with some economists maintaining that government deficits, while inflationary, displace private investment, while other, more traditional Keynesian economists claim that government deficits and monetary expansion can have real effects without at the same time inducing inflation.

The object of this paper is to establish a set of propositions concerning the circumstances under which

(a) public financial policy is irrelevant: it has neither real nor inflationary effects;

(b) public financial policy has price effects, but no real effects (as in the classical dichotomy); and

(c) public financial policy has real effects.

Two basic premises underlie our analysis: that the effects of all the financial policies of the government--both its debt and tax policies--need to be taken into account simultaneously; and that these effects can only be analyzed within intertemporal models with explicit assumptions about the formation of expectations by individuals and about the impact of the financial policy on the intertemporal distribution of income.

In Section II, I present the Basic Irrelevance Theorem, establishing that if the government's debt-cum-tax policy does not involve any intergenerational redistribution, government financial policy not only has no effect on any real variable in the economy, but it also has no effect on any financial variable (including the price level). The increase in the supply of debt (accompanying the decrease in taxes) leads to a precisely identical increase in the demand for government debt. (Accordingly, I shall sometimes refer to this result as Say's Law of Government Deficits.)

This result is in sharp contrast to the implication of deficits in the portfolio balance approach (e.g., of Tobin), though like Tobin (and unlike much of the recent literature in the new Classical macro-economics) we have explicitly assumed that all individuals are risk averse. In the portfolio balance models, the increase in government debt has real effects because individuals will not hold the additional government debt unless the return to debt relative to equities changes; but in these models, individuals are myopic--they fail to take into account future tax liabilities,[1] and when they do so, their optimal portfolio turns out to require an increase in government debt just equal to the current increase in supply.

The model of Section II involves a single, infinitely-lived generation.[2] In contrast, in the remainder of the paper we focus our attention on models with overlapping generations (and without bequests). In Section III we show

1. Thus, our result can be viewed as an extension of the Ricardo-Barro approach to include uncertainty; obviously, in the absence of uncertainty, the form in which individuals hold their assets is not of much interest; all assets are perfect substitutes.

2. Or equivalently, families, all of whom have and care about their descendant(s), with descendants, all of whom have and care about their descendants, etc. See Barro (1974).

that there exist some financial policies (in particular, an increase in the nominal interest rate paid on government debt, financed by the issuance of additional debt) which have no real effects, including no effect on the intergenerational distribution of income; at the same time, this policy does have an effect on the rate of inflation. As a result, I sometimes refer to this Second Irrelevance Theorem as establishing the neutrality of inflation.[1]

(It should be emphasized that not all inflation is of the "pure" form described in Section III. There are often other, accompanying changes in policy which have real effects.)

Most changes in public financial policy do, however, have consequences for the intertemporal distribution of income, and in Sections IV and V, I show that when the government's financial policy does involve intergenerational redistribution, then (even restricting ourselves to policies with the same expected rate of inflation) it has real effects on the economy, except in certain limiting cases. It is easy to see why alternative financial policies have an important effect on the intertemporal distribution of income. If the government should decide at some date to increase the supply of government bonds more than it had previously planned, it will increase the price level; owners of debt (the "old" in the typical life cycle model) become worse off; similarly, if it decides to decrease the debt, the price level falls, making the older generation better off, at the expense of the younger generation. In this sense, there is a close link between debt policy

1. This result thus represents an extension and generalization of an earlier result reported in Stiglitz (1981).

on the one hand, and social security policy on the other (a link which was extensively discussed in Atkinson and Stiglitz (1980) for non-stochastic models). These redistribution effects of debt policy will, in general, have a real effect on the patterns of capital accumulation; only if the demand for capital were independent of wealth would there be no such effect, a possible but implausible case.

That there is a close relationship between debt policy and capital accumulation can be seen from a slightly different perspective. It is well known, from the literature on money, debt and growth, that, in the absence of uncertainty, debt policy has a significant effect on capital accumulation.[1]

There, debt policy (the rate of change in the money or debt supply) has real effects, because individuals substitute government debt for capital in their portfolios. In those models, since there was no uncertainty, the real return on money had to equal the real return on capital,[2] and this asset equilibrium (or portfolio balance) condition determined the rate of change of prices. In the analysis here alternative debt policies change the probability distribution of the returns to financial assets (relative to, say, capital), and thus again there is a substitution between capital and government debt. Only if individuals are risk neutral--and so are in-

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1. Though earlier studies of Tobin (1965) or Shell, Sidrauski, and Stiglitz (1969) are open to the criticism that the individuals are not explicitly maximizing their intertemporal utility, the studies of Cass and Yaari (1967) and Diamond (1965) made it clear that similar results also obtained in the life cycle models. See also Atkinson-Stiglitz (1980).

2. This is a slight simplification. While Tobin (1965), Johnson (1966) and Sidrauski (1967), for instance, did not explicitly introduce uncertainty into their analysis, they treated the two assets as imperfect substitutes, without formally explaining why this was so.

different among financial policies which generate the same mean rate of inflation--can such changes in policy have no effect.

It is important to realize, however, that this is just an intuitive argument: it would appear to be equally valid for the overlapping generations (life cycle savings) model as it would for the model with long-lived individuals; yet for the latter, we establish in Section II that changes in financial policy of the government (without redistributive effects) have no real effects; the reason for this is in fact that they have no effects on prices. The price distribution is clearly endogenous, and whether it is or is not affected by a particular financial policy is the central question with which we are concerned.

Having established that, in general, public financial policy does matter, the next natural question is, what do optimal public financial policies look like? Section VI characterizes the optimal policy of intertemporal risk-sharing and income distribution, and shows that this policy can be implemented by means of a simple set of public financial policies. In Section VII we expand the set of financial policies considered so far to include debt instruments of varying maturity. We show that, in our simple model of identical individuals, the additional instruments are redundant. If, however, there are restrictions on the set of admissible taxes and individuals differ, then the additional instruments may not be redundant. Section VIII summarizes several directions in which the analysis may be extended.

Before beginning our formal analysis, there are two caveats concerning what I mean by public financial policy that I should mention. First, throughout the analysis, I keep the level of real government expenditure at each date fixed. Financial policy is simply concerned with the manner in which those real expenditures are financed (and with the inseparable question of how income is redistributed among individuals). Second, I am not concerned here with those issues arising from there being both interest and non-interest bearing short-term financial assets in the economy at the same time. (I have dealt with those issues extensively elsewhere (Stiglitz (1982).) I shall focus extensively on the demand for financial assets as a store of value.[1,2]

A standard question that is often raised at this juncture, in our argument that public financial policy is[3] relevant, is what can the government do that the private sector cannot do (or undo)? Within the life cycle model, there are two answers: first, the government can engage in intergenerational redistribution, which the private sector cannot undo; second, by the very structure of the model, there cannot exist a full set of Arrow-Debreu securities in such an economy: there is no way that individuals at date t can trade the risks which they face with individuals at date 0. Government financial policy can provide risk-sharing opportunities which the private market cannot provide.

II. The First Irrelevance Proposition: Say's Law of Government Deficits

In this section, we develop a simple model in which debt policy has neither real nor financial effects. We consider an economy with infinitely-

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1. Recent developments in financial markets make it clear that the costs involved in providing transactions services associated with interest bearing bonds is not significant.

2. I shall occasionally refer to government policies with respect to the supply of short-term bonds (or more general financial policies) as "monetary" policies.

3. With the minor exceptions previously noted.

lived individuals, with wages at time t in state $\theta(t)$ of $w_i(t,\theta)$, lump sum taxes or transfers of[1] $T_i(t,\theta)$, consumption of $C_i(t,\theta)$, labor supply of $L_i(t,\theta)$, holdings of capital[2] of $K_i(t+1, \theta(t))$, and holdings of the single, interest bearing financial asset of $B_i(t,\theta)$. In the absence of uncertainty $\{L_i,C_i,K_i,B_i\}$ are chosen to maximize the individual's lifetime expected utility, which can be expresses simply as a function of the vectors $\{\underline{L}_i,\underline{C}_i\}$:

$$U_i = \{U_i \; \underline{L}_i,\underline{C}_i\} \quad ,$$

subject to the lifetime budget constraints.[3]

For simplicity, we take consumption as our numeraire; we assume that the price ratio of capital goods to consumption goods is fixed at unity (this, like the assumption of a single consumption good, is a simplifying assumption which can easily be removed). We let $v(t,\theta)$ be the price of bonds in terms of consumption goods; $p(t,\theta) \equiv \frac{1}{v(t,\theta)}$ is the price of goods in terms of the financial asset. We shall refer to p as the price level. Let $\rho(t,\theta)$ be the real rate of return on a financial asset. In general, this consists of two parts, an interest payment and a capital gain (or loss). If $i(t,\theta(t))$ is the interest paid at date $t+1$ on a bond purchased at date t in state $\theta(t)$,[1] then

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1. We shall, for notational simplicity, simply write θ for $\theta(t)$ when there is no ambiguity.

2. $K_i(t+1,\theta(t))$ is the amount of capital purchased at time t, but used at date $t+1$.

3. It should be clear that nothing in this formulation requires us to restrict our analysis to preference orderings satisfying the expected utility axioms.

$$(1) \qquad \rho(t,\theta(t),\theta(t+1)) = \frac{i(t,\theta(t))}{v(t,\theta(t))} + \frac{v(t+1,\theta(t+1))}{v(t,\theta(t))} - 1$$

Clearly, in the absence of uncertainty, if the <u>marginal</u> transactions value of the financial asset is zero,[2] and η is the real rate of return on capital,[3]

$$(2) \qquad \eta(t) = \rho(t)$$

the real return on bonds must equal the real return on capital.

The value A_i (in real terms) of the individual's assets at time t in state $\theta(t+1)$ is

$$(3) \qquad A_i(t,\theta(t)) = K_i(t,\theta(t-1))(1+\eta(t,\theta(t))$$

$$+ \; B_i(t-1,\theta(t-1))[v(t,\theta(t)) + i(t-1,\theta(t-1))]$$

$$+ \; w_i(t,\theta(t))L_i(t,\theta(t)) - T_i(t,\theta(t))$$

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1. It makes no difference for our analysis whether the individual knows the interest which will be paid, i.e., $i = i(t,\theta(t))$ or does not, i.e., $i = i(t,\theta(t+1))$. We shall, for simplicity, assume he does. i is a <u>real</u> interest payment, i.e., i is measured in terms of consumption goods. This assumption too is made for notational simplicity. It is more realistic if we let i be dominated in financial units, so

$$\rho(t,\theta(t),\theta(t+1)) = (1 + i(t,\theta(t))\frac{v(t+1,\theta(t+1))}{v(t,\theta(t))} - 1$$

The requisite modifications to the analysis are straightforward. Even if i is measured in consumption goods, and is specified at t, ρ is uncertain because $v(t+1,\theta(t+1))$ is uncertain. In Stiglitz (1982) the analysis is extended to indexed bonds.

2. As we assume throughout this paper. But see Stiglitz (1982).

3. η will, in general, depend on the capital-labor ratio. The individual, however, simply takes η as given, and hence in our notation, we simply write η as a function of t (and θ). Again, when there is no ambiguity, we suppress the dependence of variables (such as ρ and η) on θ.

i.e., the capital he had at the end of the last period, plus the return on that capital, plus the value of his bonds, plus the interest payments and wage payments, minus lump sum taxes. This wealth can be used to purchase goods or assets,[1] i.e.,

$$A_i(t,\theta(t)) = C_i(t,\theta(t)) + K_i(t+1,\theta(t)) + v(t,\theta(t))B_i(t,\theta(t)) \quad . \tag{4}$$

We assume a neoclassical production function of the usual form,

$$F(K,\ L,\ \theta) = C + \Delta K + G \quad , \tag{5}$$

where G is expenditure on public goods, K is aggregate capital, C is aggregate consumption, L aggregate labor supply:

$$K = \Sigma K_i \tag{6a}$$

$$L = \Sigma L_i \tag{6b}$$

$$C = \Sigma C_i \quad . \tag{6c}$$

Market equilibrium requires, in addition, that if $B^*(t,\theta)$ is the outstanding government debt at time t

$$B^*(t,\theta) = \Sigma B_i(t,\theta) \quad . \tag{7}$$

The demand for bonds must equal the supply of bonds. Moreover, we require real government revenues (taxes plus revenues from the issue of new bonds)

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1. In a finite period model, we have a natural boundary condition

$$K_T = B_T = 0 \quad .$$

In the infinite period problem, we need to impose a corresponding transversality condition.

to equal real government expenditures (interest payments plus purchases of public goods).[1]

(8) $$i(t-1,\theta(t-1))B^*(t-1,\theta(t-1)) + G(t,\theta(t))$$

$$= v(t,\theta(t))[B^*(t,\theta(t)) - B^*(t-1,\theta(t-1))] + \Sigma T_i(t,\theta(t))$$

Equation (8) is the government budget constraint.

We now establish, in this simple context, the debt neutrality proposition. Assume at t_1 the government increases $B^*(t_1)$ by one bond and (to keep the government budget constraint satisfied) decreases $\Sigma T_i(t_1)$ by $v(t_1)$. Now assume that at some later date, t_2, the government restores the debt to its previous level; again, if government expenditures remain unchanged, this necessitates an increase in taxes by $v(t_2,\theta)$. At intervening dates, to keep (8) satisfied,[2]

$$\Sigma\Delta T_i(t,\theta) = i(t,\theta) \quad .$$

Finally, let us assume that the taxes are imposed in such a way as to have no redistributive effect, i.e.,

(9) $$\frac{\Delta T_i(t,\theta)}{\Sigma\Delta T_j(t,\theta)} = \frac{\Delta T_i(t_1)}{\Sigma\Delta T_j(t_1)} \qquad \text{all } t,\ i,\ \text{and } \theta$$

Corresponding to this new tax-debt policy, there exists a new equilibrium in the private sector. Denote by a single caret the original equilibrium

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1. If all the other equilibrium conditions are satisfied, (8) must be satisfied.

2. We could, alternatively, finance the additional interest payments by additional bond issues; if these new bond issues are themselves retired at t_2, the analysis remains unchanged.

values, and by a double caret the new equilibrium values. Then

$$
(10)\qquad
\begin{aligned}
\hat{\hat{C}}_i(t,\theta) &= \hat{C}_i(t,\theta)\\
\hat{\hat{L}}_i(t,\theta) &= \hat{L}_i(t,\theta)\\
\hat{\hat{K}}_i(t,\theta) &= \hat{K}_i(t,\theta)\\
\hat{\hat{v}}_i(t,\theta) &= \hat{v}_i(t,\theta) \quad \text{or, equivalently,} \quad \hat{\rho}(t,\theta) = \hat{\hat{\rho}}(t,\theta)\\
\hat{\hat{B}}_i(t,\theta) &= \hat{B}_i(t,\theta) \quad \text{for} \quad t < t_1 \quad \text{and} \quad t > t_2\\
\hat{\hat{B}}_i(t,\theta) &= \hat{B}_i(t,\theta) - \frac{\Delta T_i(t_1)}{v(t_1)} \quad \text{for} \quad t_1 \le t \le t_2 \quad .
\end{aligned}
$$

To see this, assume that all aggregate variables other than $B^*(t,\theta)$ remain unchanged.[1]

From (3) and (4)

$$
(11)\qquad
\begin{aligned}
\hat{C}_i(t,\theta(t)) &= K_i(t,\theta(t-1))(1+\eta(t,\theta(t))\\
&+ \hat{B}_i(t-1,\theta(t-1))[\hat{v}(t,\theta(t)) + \hat{i}(t-1,\theta(t-1))]\\
&+ \hat{w}_i(t,\theta(t))\hat{L}_i(t,\theta(t)) - \hat{T}_i(t,\theta(t))\\
&- \hat{K}_i(t+1,\theta(t)) - \hat{v}(t,\theta(t))\hat{B}_i(t,\theta(t)) \quad .
\end{aligned}
$$

It is apparent from (9) and (10) that the policy described by (10) is feasible, and yields exactly the same consumption profile over time as did the original equilibrium. In fact, the feasible set of consumptions for

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1. In the new situation, there may, of course, be more than one equilibrium, just as in the old situation there may be more than one equilibrium. The argument is only that corresponding to any equilibrium in the original situation there exists an equilibrium in the new situation which is related to the original equilibrium by equation (10).

each individual in the new situation is identical to that in the old,[1] and hence each individual will choose exactly the same values of consumption, capital holdings, and labor supply in each state and at each date, and will only alter his bond holdings in the manner indicated. But, if they do this, the increase in the demand for bonds will precisely equal the increase in the supply of bonds. Hence, if all markets cleared before, they do now. Debt policy has no effects on either the real economy or on the price level.

The proof we have employed is a straightforward extension of the proof I used earlier to establish the irrelevance of corporate financial policy (Stiglitz (1969, 1974).)[2,3] The critical assumption in that analysis was that of no-bankruptcy. Here, bankruptcy is not an issue,[4] since the govern-

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1. That is, any consumption-labor sequence which is feasible in the new situation is feasible in the original situation, and conversely. This ignores any non-negativity constraints. See below and Stiglitz (1982).

2. See also Atkinson-Stiglitz (1980).

3. For another application of this kind of analysis, see Wallace (1981). His model differs in a number of important ways from that presented here. In particular, he employs a life cycle model, and in his model there is a complete set of Arrow-Debreu securities. He focuses his attention on changes in financial policy which are accompanied by changes in government's holding of capital, and thus are not pure financial changes (as we have defined that term).

4. The reason that bankruptcy made a difference in the earlier analysis was that it resulted in the creation of a new security; in the absence of a complete set of Arrow-Debreu securities, this, of course, may have real effects. Similarly, public financial policy--the issuance of a new kind of bond--may result in the creation of a new security, in the absence of a complete set of Arrow-Debreu securities. But the simple kinds of financial policy considered in this section cannot have that effect. But see Section VII below and Stiglitz (1982).

   There is a sense in which bankruptcy is relevant here too: as we note below, the proof of the irrelevance proposition requires that as the individual borrows more, the interest rate he has to pay is unchanged. This will be the case only if there is no probability that the borrower defaults.

ment can always impose taxes to pay back the bonds. What is critical here is the assumption that the bonds will eventually be redeemed, that there are no distributive implications of the tax changes (equation (9)), and that there are no binding constraints on individual borrowing.

Note that in obtaining this result we did not specify how the bonds were distributed by the government to the private sector. The changes in taxes that the government must undertake, if it is, at the same time to issue more bonds while keeping real expenditures fixed, in conjunction with the anticipated increases in taxes at some later date associated with the subsequent retirement of the new bonds, generate precisely the requisite demand for bonds. The only direct action of the government is to change taxes and to change the supply of bonds publicly offered. The market takes care of the rest. (Thus, this result is quite different from that associated with "money rain" or, in this context, "bond rain.")

Note too that although we have assumed that the bonds will be retired at a particular date t_2 , the retirement date itself can be a policy variable, a function of θ . So long as individuals anticipate that the current deficits will eventually be retired by the imposition of taxes in the future, debt policy has no real effects and is non-inflationary.[1]

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1. Some difficult problems arise if there is some probability that the government will never retire the bonds. Then the increase in the government debt is inflationary. To see this, assume that prices remain unchanged and that individuals' consumption, labor supply and capital holdings remained unchanged at each date and in each state. Then their bond holdings must have increased, and if the transversality condition held before, it no longer holds. Assume now that v falls in proportion to the increase in B , so that vB remains unchanged, and that the government reduces i proportionately at each date. Then, it is easy to show that nothing real has changed, and hence if we were initially in equilibrium, we will still be in equilibrium, with p increasing proportionately at each date and state. Now, if we move from this equilibrium to a new equilibrium where i remains at its original value, but the corresponding differences in government expenditure are reflected in changes in new issue of government bonds, then we again obtain an equilibrium in which all real variables remain unchanged, but the price level has changed. (See below, Section III.)

One might be tempted to argue that if nothing real changed, as we have asserted, then individuals will want to allocate their portfolios in the same ratios between bonds and capital (if, say, the individual had constant relative risk aversion) as before; but since the relative supply of bonds has increased, this implies that the market could not be in equilibrium with real variables unchanged. This argument ignores the nature of the tax liability which the individual anticipates will be imposed on him in the future. The individual hedges this particular risk by holding on to bonds (since he knows that the magnitude of the tax liability will be related to the price of the bonds by the basic government budget constraint). There is a simple moral to this story: traditional portfolio theory, based on myopic risk analysis, may be seriously misleading when analyzing intertemporal equilibrium.[1]

We summarize this section in Proposition I. (The General Irrelevance Theorem, or Say's Law of Government Deficits). An increase in the government deficit has neither real nor inflationary effects so long as the associated changes in taxes are distribution neutral and so long as the debt will eventually be reduced to its original level.

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1. Another area in which myopic portfolio analysis has recently been shown to be very misleading is in the analysis of the effect of capital gains taxation. A reduction in the tax rate on long-term capital gains might increase government revenue, but at the same time lead to an increase in consumption and a reduction of savings (since individuals' future tax liabilities have been reduced.) See Stiglitz (1981).

III. The Second Irrelevance Proposition: The Neutrality of Inflation

In this section we prove a second irrelevance proposition. We establish Proposition II. A change in the interest rate paid (in any state of nature, at any date) financed by an increase in debt has an effect on the price level, but not on any real variables. In particular, the real value of debt (Bv) at all subsequent dates and states remains unchanged.

This proposition is true not only in models with infinitely-lived individuals, but also in life cycle models. An immediate corollary of this proposition, then, is that such a financial policy has no intergenerational distributive implications.

Since in the subsequent sections of this paper we shall focus our attention on the life cycle model, we establish the proposition in that context. The modifications required to establish the proposition for the economy analyzed in the previous section are straightforward.

We assume individuals live for two periods, working in the first, and saving part of their wage income for their retirement. In the subsequent discussion, all variables are functions of t and the state of nature, but for notational simplicity we shall suppress the dependence on the state of nature except where it would give rise to ambiguities. To simplify our analysis further, we assume labor is inelastically supplied, with L normalized at unity. We assume a constant population, which we also normalize at unity.

For simplicity, we assume that individuals' utility functions are separable.[1,2]

(12) $$U = u_1(C_1(t)) + \beta\, u_2(C_2(t))$$

where

$C_1(t)$ is the consumption the first period of those born at date t, and

$C_2(t)$ is the consumption the second period of those born at date t.[3]

The individual maximizes his expected utility

(13) $$\max E\, u(C_1(t)) + \beta\, u(C_2(t))$$

subject to this budget constraint, which we can write in parametric form as

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1. This plays no role in this section, but has some interesting implications for the analysis of Section VI.

2. As in the standard life cycle model, we assume away altruism: individuals do not care either for their antecedents or their descendants. If all individuals care about their children, and their children care about their children, then clearly, we obtain a derived utility function where consumption at all future dates enters into the individual's welfare function (see Barro (1976)). Though the fact that some individuals do intentionally leave bequests suggests that the assumption of no altruism is extreme, the assumption that everyone leaves a bequest, and adjusts fully for a change in government debt by a change in his bequest, is also extreme. The qualitative propositions presented in this section require only that there exist some individuals who leave no bequests, either because of a complete lack of altruism, or because they have no children. Since, in fact, a significant fraction of the population has no children, and with a non-zero probability, any individual will have only a finite number of descendants, we believe that the qualitative results presented here are of some relevance.

3. Thus, $C_2(t)$ occurs at date $t+1$.

$$(14) \qquad C_2(t) = K(t+1)(1+\eta(t+1)) + B(t)(v(t+1) + i(t)) - T_2(t)$$

$$(15) \qquad K(t) + B(t)v(t) = w(t) - T_1(t) - C_1(t)$$

where

$T_1(t)$ = lump sum taxes on young individuals at date t , and

$T_2(t)$ = lump sum taxes on old individuals at date $t+1$.

Equation (15) simply says that the individual takes the resources available to him at date t (his wages minus lump sum taxes), and either consumes them or saves them; and if he saves them, he saves them either in the form of bonds or in the form of capital (equities). Equation (14) says that the individual's consumption the second period of his life consists of his return on capital and bonds, minus any lump sum taxes, plus what he can sell his capital and bonds for to the younger generation. (These are just equations (3) and (4) rewritten for this simple case.)

Individuals form expectations concerning future prices and are assumed to know the probability distribution of the return on equities. They are also assumed to know the probability distribution of the real lump sum transfers that they will receive when they are old. This yields, in a straightforward way, individuals' optimal consumption and investment decisions:[1]

$$(16a) \qquad C_1(t) = C_{1t}(w(t) + T_1(t), \tilde{\rho}(t), \tilde{\eta}(t+1), \tilde{T}_2(t))$$

$$(16b) \qquad K(t+1) = K_{t+1}(w(t) + T_1(t), \tilde{\rho}(t), \tilde{\eta}(t+1), \tilde{T}_2(t))$$

---

1. $C_2(t)$ is determined as a residual, from equation (14).

$$(16c) \qquad v(t)B(t) = B_{t+1}(w(t) + T_1(t), \tilde{\rho}(t), \tilde{\eta}(t+1), \tilde{T}_2(t))$$

(Equations (16a), (16b), and (16c) are, of course, not all independent; from the budget constraint, knowing $C_1(t)$ and $K(t)$ we can infer what $B(t)$ must be. Equation (16c) shows the important property that the real demand for bonds (vB) depends on the real rates of return on the different assets.)

Assume we initially have an equilibrium, with the values of all (market clearing) variables denoted by a single caret. Assume now that at t_1 the government increases $\hat{i}$ to $\hat{\hat{i}}$ and finances the increased interest payments by issuing more bonds. Then, there exists a new equilibrium to the economy with all real variables unchanged, but with (denoting the new equilibrium values by double carets):

$$(17) \qquad \frac{\hat{\hat{v}}(t+1) + \hat{\hat{i}}(t)}{\hat{\hat{v}}(t)} = \frac{\hat{v}(t+1) + \hat{i}(t)}{\hat{v}(t)} = \rho(t) + 1 \quad \text{for all} \quad t \geq t_1 \ .$$

$$\hat{\hat{v}}(t) = \hat{v}(t) \quad \text{for} \quad t < t_1 \quad ,$$

i.e., the rate of inflation will adjust to keep the real return on debt the same (in every state of nature) and at every date. Moreover,

$$(18) \qquad \hat{\hat{v}}(t)\hat{\hat{B}}(t) = \hat{v}(t)\hat{B}(t) \quad \text{for all} \quad t \ .$$

Since the real returns on all assets are unchanged, and taxes are unchanged, demands for capital, "real" bonds (vB) and consumption are unchanged. If all markets cleared in the initial situation, they still do.

To confirm that the government budget constraint is satisfied, we rewrite (8) using (1):

$$(19) \qquad G(t,\theta(t)) = \Sigma T_i(t,\theta(t))$$
$$+ \; v(t,\theta(t))B^*(t,\theta(t)) - v(t-1,\theta(t-1))B^*(t-1,\theta(t-1))$$
$$+ \; \rho(t,\theta(t),\theta(t-1))v(t-1,\theta(t-1))B^*(t-1,\theta(t-1)) \quad .$$

So long as vB and ρ remain unchanged, the government's budget constraint will be satisfied at each date, in every state. It immediately follows that any <u>sequence</u> of such changes (such as a permanent change in the interest rate) also has no effect on the economy.

IV. The Fundamental Relevance Theorem

In the preceding two sections, we provided two general sets of conditions under which government financial policy would not matter; in Proposition I, it had neither real nor financial (price) effects, while in Proposition II, it had no real effects, but there were effects on the price of bonds (relative to consumption goods). In this section, we show that changes in financial structure--other than those described in the preceding two propositions--always have a real effect on the economy.

Not surprisingly, it makes a difference to the analysis whether the change in financial policy is announced (or anticipated) or unannounced (unanticipated). We first consider the effects of perfectly anticipated policy changes. Assume that the government announces that at some date, t_1, in the future, it will increase its debt, and at the same time changes T_1 and T_2 to keep the budget constraints of the government satisfied. At some subsequent date, t_2 $(t_2 > t_1 + 2)$, it will decrease the outstanding debt and increase taxes in a corresponding way. In intervening periods, it increases bonds to pay the additional interest costs. (This is the kind of policy change we considered in Section II, but there, the individuals were infinitely-lived, so there was no intergenerational distributive effect of the change.) Such a change obviously affects the consumption of different individuals. The question is, under what conditions will the change in debt policy have no real aggregate effects, e.g., on the level of capital accumulation?[1]

There is one special case that we shall focus on, that will be helpful in developing our intuition concerning the nature of the equilibrium. Assume that there is no risk, or that individuals are risk neutral. Clearly, as we noted earlier, bonds and capital must yield the same return and they will then be perfect substitutes.

The argument that the financial change we described above will have real effects is simple. Either the price of bonds on the new path is the same as on the old path, or it is not. Assume the prices are the same. Then, clearly, real returns at each date are unchanged. For this to be an equilibrium, individuals at all dates from t_1 on must be willing to hold the larger (real) bond supply. But an individual born at any date after t_1 and dying at any date before t_2 finds his budget constraint unaffected, and thus has his wage income, lump sum transfers, and savings unaffected. If real capital accumulation is to be unchanged, therefore, his holdings of real bonds must be unchanged, contradicting our assumption that the real bond supply has increased.

Suppose now that prices change. For simplicity, assume i is unchanged. (The Second Irrelevance Theorem implies that this makes no difference.) Rewriting (15)

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1. Or, if labor were elastically supplied, on the level of employment.

$$B(t)v(t) = [w(t) - C_1(t) - K(t+1)] - T_1(t)$$

This implies that if the policy change is to have no effects on $K(t)$[1]

$$\hat{v}(t) \equiv \hat{\hat{v}}(t) \quad \text{for} \quad t < t_1-1,\ t > t_2+1$$

and

$$\hat{v}(t)\hat{B}(t) \equiv \hat{\hat{v}}(t)\hat{\hat{B}}(t) \quad \text{for} \quad t_1+1 \leq t < t_2-1$$

This implies that over the interval $(t_1-2,\ t_1+1)$ the average rate of return on bonds must have been less than that on capital, and hence this could not be an equilibrium.

There are two conditions imposed on the equilibrium; one relating to the equality of the returns between financial assets and capital assets, the other that investment must be equal to savings minus holdings of financial assets. It is impossible, within the life cycle model, to change the supply of bonds in such a way as to have no real effects. Only if the bond supply is increased at t_1 and retired at t_1+1, and the additional revenues generated at t_1 are used to finance a lump sum subsidy to the young, while a lump sum tax is levied on the old at t_1+1 to retire the debt, is the financial policy neutral. But then, the financial policy affects only the t_1 generation, i.e., it is completely described by Proposition I.

It should be clear that the assumptions of risk neutrality or no risk, though they simplified our exposition, were not critical to the results.

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1. That is, for $t < t_1-1$ and $t > t_2+1$, $\hat{B} = \hat{\hat{B}}$ and $\hat{T}_1 = \hat{\hat{T}}_1$.

Even if the increase in the government debt at t is unanticipated, the policy change will have a real effect unless it is anticipated that there will be no subsequent reduction in the government debt. So long as an unanticipated change gives rise to anticipations of further changes, the previous analysis (<u>mutatis mutandis</u>) applies.

This analysis has one interesting corollary. Assume for simplicity that the single financial asset is non-interest bearing ($i = 0$). Assume, moreover, that the government announces that it will increase the bond (money) supply by a given percentage. It is sometimes supposed that equilibrium will be restored simply by an equi-percentage reduction in the price of bonds (so that the real bond supply remains unchanged). But if this change were anticipated, it would have had effects on the demand for bonds in preceding periods. Only if individuals completely ignore the asset return will such a change be neutral. Moreover, if it is believed that the increase in the bond supply is temporary, with it returning to the previous level the next period, with prices at subsequent dates unaffected, individuals will now anticipate a larger return to holding the bond than they obtained previously, and this will induce them to hold more bonds, again contradicting the assumption of no real effects. On the other hand, if it is believed that the increase in the bond supply is permanent, unless in the previous situation the bond supply at all future dates were fixed, then the fixed increase in bond supply represents a variable proportionate increase. Thus, for the real bond supply to be fixed at every date requires the return to the bond to vary from date to date. Finally, even if it is believed that there will be an equi-proportionate increase in the bond supply at every date, so that if the price of bonds fell by a given percentage, the real bond supply at each date would be unchanged, there will be real effects. If it were anticipated, of course, it would have had real effects in previous periods. But even if it is unanticipated, it will have real effects, through the government's budget constraint. The equi-proportionate fall in the price of bonds is equivalent to a lump sum levy on the present owners of bonds. Only if the extra revenue generated by this "tax" is spent on the old (the owners of the bonds) will there be no distributive effects of the change (and hence will there be no real effects).

We can summarize the results of this section on Proposition III. <u>Any anticipated changes in financial action other than those described in Propositions I and II, have real and financial effects on the economy</u>.

<u>Any unanticipated change has no real effects on the economy only if (a) it does not give rise to anticipations of further changes</u> (i.e., it does not change individuals' subjective probability distributions concerning future government actions); <u>and (b) increases in debt are used to provide lump sum subsidies to current owners of the financial asset</u> (the aged).

These results should not be surprising: it is well known that in this form of the simple life cycle model there are simple equivalency relationships between debt policy and social security policy; they induce equivalent intergenerational redistributions of income and will, in general, have real effects. (See Atkinson-Stiglitz, 1981.) We shall return to this theme in Section VI.

V. <u>Second Relevance Proposition</u>

So far, we have considered the effect of a change in the financial structure of the government at two points of time. We saw how any such changes would have real effects. We now ask, are there combinations of such changes, with offsetting real effects? In particular, we now consider the effect of financial policies, i.e., rules that specify what the government will do under each contingency. In our simple model, the government controls four variables; the bond supply, the interest it pays on government debt, and the lump sum transfers to the young and to the old. It can make these variables a function of all observable variables, i.e., letting

$$x(t) = \{K(t), L(t), C_1(t), C_2(t-1), \psi(t), v(t)\}$$

where $\psi(t)$ is the vector $\{\eta(t), w(t)\}$, the exogenous variables describing the state of the economy at any time, and

$$x^*(t) = \{x(t), x(t-1), \ldots\}$$

i.e., the entire history of the observable variables up to and including their values at date t , then a government financial policy is a sequence of functions[1] of the form

$$\text{(20)} \qquad T_1(t) = T_{1t}(x^*(t))$$

$$T_2(t) = T_{2t}(x^*(t))$$

$$B(t) = B_t^s(x^*(t))$$

$$i(t) = i_t(x^*(t))$$

which satisfy the government's budget constraints. Thus, future government actions are unknown, simply because the events on which they will be based

1. In this formulation, actions at date t depend on observables at t . Other formulations, with lags in observations, will work as well.

are unknown; but the policies are assumed to be known. As soon as the events on which they depend become known, the government action is well specified.

A rational expectations equilibrium can now be easily defined (for each set of feasible policy functions). For each public financial policy (set of functions (20)), and for each set of expectations about the price distribution

$$v^e(t) = v^e(x^*(t-1), \eta(t), w(t))$$

(prices next period are a function of the entire history of observables up to and including their values at $t-1$ and the realization of the exogenous variables η and w), there will be a demand for bonds $B_t^d(x^*(t))$. Equilibrium requires that this demand for bonds equal the supply

$$B_t^d(x^*(t)) = B_t^s(x^*(t)) \quad \text{for all } x^*(t) \quad . \tag{21}$$

Rational expectations requires in addition that given the assumed known probability distributions of η and w , and the policy functions (20), expectations are realized

$$v^e(x^*(t)-1), \eta(t), w(t)) \equiv v(x^*(t-1), \eta(t), w(t)) \quad . \tag{22}$$

A simple policy, for instance, would be to increase the bond supply by x% if the return to capital exceeds its average value, decrease it by x% if the return to capital is less than its average value. This kind of rule makes little sense. In a sequel to this paper (Stiglitz (1982)), we consider the consequences of several simple but more reasonable rules.[1] For now, we wish to show that, even if the government restricts itself to policies which are functions of current exogenous variables, and confines itself to policies which, in any state, have the same expected rate of inflation, i.e., $E\hat{\rho} = E\hat{\hat{\rho}}$, changes in financial policy have real effects.

The government, for instance, announces that if, at t, $w(t) = w_1$, it will increase the bond supply more than it had planned to do under the original financial policy, while if $w(t) = w_2$, it will increase the bond supply less. The two changes are chosen so that, in the rational expectations equilibrium, the expected rate of change in the price level is unchanged.

Consider first the case where individuals are risk neutral; by our earlier analysis we can, without loss of generality, restrict ourselves to economies in which government bonds pay no interest, so in equilibrium[2]

$$(23) \qquad Ev(t+1) = v(t)(1+\bar{\eta}) \quad .$$

We investigate two cases, that where the changes in bonds are accompanied by changes in taxes on the young, and that in which they are accompanied by by changes in taxes on the old. In the latter case, since $T_1(t_1)$ is unchanged,

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1. A specification of a financial policy requires specifying not only the circumstances under which, for instance, B is increased or decreased, but who is taxed or subsidized. We consider three alternative rules for deficits (keeping the bond supply constant, keeping prices constant, keeping the real bond supply, vB, constant) under the assumption that any resulting deficits (or surpluses) are financed by (distributed as) lump sum taxes (subsidies) on, alternatively, the young or the old.

2. If we restrict ourselves to economies in which bonds pay an interest of $E\eta(t)$, so equilibrium requires

$$Ev(t+1) = v(t),$$

i.e., the price level is a Martingale.

if $K(t)$ were unchanged for all t , vB would be unchanged for all t . This follows from substituting the government's budget constraint into the individual's budget constraint, to obtain

$$(24) \qquad v(t)B(t) + T_1(t) = w(t) - C_1(t) - K(t+1) \quad .$$

If $T_1(t)$ is unchanged, individuals' opportunity sets are unchanged,[1] and hence $C_1(t)$ is unchanged. But this implies that $T_2(t_1-1)$ must be increased. Rewriting the government's budget constraint for this case, we obtain (from (8)

$$(25) \qquad G(t,\theta(t)) = v(t,\theta(t))B(t,\theta(t)) - \frac{v(t,\theta(t))}{v(t-1,\theta(t-1))} \times$$

$$v(t-1,\theta(t-1))B(t-1, \theta(t-1)) + T_2(t-1,\theta(t)) \quad .$$

Hence, if $B(t_1,\hat{\theta}(t_1))$ is changed for some $\hat{\theta}(t_1)$, $v(t_1,\hat{\theta}(t_1))$ must have changed, and hence $T_2(t_1-1,\hat{\theta}(t_1))$ must have changed. If, however, the expected value of $v(t_1)$ is unchanged,(i.e., (23) is satisfied), the expected value of $T_2(t_1-1)$ is unchanged, and if individuals' behavior only depends on their expected taxes next period, this change has no effect on capital accumulation at dates prior to t_1 . Under these circumstances, then, this change in financial policy has no aggregate real effects. (By the same token, a sequence of such changes, e.g., changes in the financial rules at every date, or at the same date in more states, will have no real effects.)

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1. We required in addition that $T_2(t)$ be unchanged for $t \geq t_1$. But if vB and ρ at all subsequent dates (in all states) are unchanged, then the government's budget constraint will be satisfied, without the alteration in any taxes, and in particular, without the alteration in $T_2(t)$ for $t \geq t_1$.

But this is, essentially, the only circumstance in which a change in financial policy has no real effects. Consider, by contrast, what happens if the change in the debt is accompanied by a change in T_1 . From the government's budget constraint, it is clear that an increase in the return on government bonds in some state 0 accompanied by a tax on the young is equivalent to a transfer of resources in that state from the young to the old. But the marginal propensity to consume of the old is unity; the marginal propensity to consume of the young is, in general, less than unity; and hence the total demand for consumption goods increases. But then it is impossible for the level of capital accumulation in that state to remain unchanged.

Moreover, even if the government accompanies changes in the returns to government bonds by changes in the taxes of the old, these changes will not be neutral if individuals are not risk neutral. For our earlier analysis showed that if $v(t_1)B(t_1)$ remains unchanged, in all states, C_2 will remain unchanged. But then individuals at t_2 will not be in portfolio equilibrium, except if the marginal utility of consumption in the two states for which ρ is altered are the same.[1] This establishes that the previous argument for the neutrality of financial policy cannot be extended to the

1. Equilibrium portfolio allocation requires

(26) $$Eu_2'(\rho-\eta)$$

where

$$u_2' = \frac{\partial u(C_2)}{\partial C_2} .$$

The condition for equilibrium savings

(27) $$u'(C_1) = Eu'(C_2)\eta$$

will be satisfied.

case of risk averse individuals.[1]

The results of this section are summarized in Proposition IV. Mean-ρ preserving changes in financial policy have no real effects if and only if the individual is risk neutral and changes in the level of debt are offset by changes in lump sum taxes/subsidies for the aged.

VI. Implementability of Optimal Intertemporal Risk Redistribution Schemes Through Financial Policies with Constant Price Levels

We have stressed in the preceding two sections that alternative governmental financial policies have real effects, largely because they generate changes in the intertemporal distribution of risk and wealth. Because individuals of different generations cannot get together to trade risks, the only way such risks can be exchanged is through governmental action. Any financial policy has implications for the intergenerational distribution of risk bearing, and changes in the financial policy thus benefit some generations at the expense of others. The government needs to take this into account when designing its financial policies.

In this section, we characterize the optimal intergenerational distribution of risk bearing and show that this policy can be implemented by means of a financial policy with constant prices, provided that the government's ability to levy lump sum taxes on the young and the old is sufficiently flexible.

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1. What this establishes is that if financial policy is to have no real effects at t_1, it must change $v(t_1)B(t_1)$. To establish that the change in financial policy must have real effects, we need to show that it is not possible for there to be a sequence of changes in $v(t)B(t)$ for $t \geq t_1$, and associated changes in $T_2(t)$ (to keep the government's budget balance), such that the level of capital accumulation at each date is unchanged. We do not present the proof here.

The problem of the optimal intertemporal allocation of resources can be easily formulated; for simplicity, we assume an additive social welfare function of the standard form

(28) $$E \sum_t \frac{u^t}{(1+\delta)^t}$$

where u^t denotes the utility of the t^{th} generation, given by

$$u^t = u_1(C_1(t)) + \beta\, u_2(C_2(t))$$

where $1/1+\delta$ is the social rate of discount. We wish to maximize (28) subject to the resource constraints of the economy. For simplicity, we assume that labor is fixed ($L = 1$) and that capital (like rabbits) can be eaten, so that the resources available at date t are given by:[1]

(29) $$S(t) = w(t) + K(t)(1+\eta(t))$$

while

(30) $$K(t+1) = S(t) - C_1(t) - C_2(t-1) \quad .$$

In addition, there is a natural non-negativity constraint on $K(t)$:

(31) $$K(t) \geq 0 \quad .$$

Formulated in this way, we have converted our problem into a standard optimal savings problem with random wages and returns on capital, with the standard non-negativity constraints on capital. This can be solved using

1. Again, for simplicity, we have ignored population growth. This may easily be incorporated into the analysis.

dynamic programming techniques.[1] Our interest here, however, is not in characterizing the solution so much as in providing an analysis of the implementation of the optimal intertemporal redistribution of income through financial and tax policy. Hence, we simply assert that the solution yields consumption and capital accumulation functions of the form[2]

$$(32) \qquad C_1(t) = C_1^*(S(t))$$

$$(33) \qquad C_2(t-1) = C_{2t-1}^*(S(t))$$

and

$$(34) \qquad K(t+1) = K_{t+1}^*(S(t)) \quad .$$

The assumptions that wages and the return on capital, at each date, are identically distributed independent random variables are essential in obtaining this simplification. If, for instance, wages were described by a random walk, then w itself would be a state variable.

We now consider how this optimal colution can be implemented in a market economy with a single financial instrument.

To implement any policy, it must control, at each date, and each state, three variables; C_1, C_2, and K . The government has four instruments, T_1, T_2 , i and B . This suggests a redundancy of instruments, and indeed,

1. The solution to this problem, ignoring the non-negativity constraints, is fairly straightforward. Taking these non-negativity constraints into account, however, complicates the problem in an essential way. Newbery and Stiglitz (1981) provide an extensive discussion of the solution of this problem for the special case where η_t is non-random.

2. The assumption of separability of the utility function was essential in arriving at this simplification in the structure of the solution. With a non-separable utility function, there are two state variables describing the economy at any date t , $S(t)$ and $C_1(t-1)$.

Proposition II showed that there was such a redundancy. We could change i and change the bond supply in such a way as to keep the government's budget balanced, and have no real effects; such a policy would, however, have an effect on the price level (v).

The four instruments are not independent, since they are linked together by the government's budget constraint. There are thus three independent instruments. On the other hand, the three variables C_1, C_2, and K are also not independent; they are linked together by the individual's budget constraint, or, equivalently, by the national income constraint:

$$C_1(t) + C_2(t-1) + K(t+1) = w(t) + (1+\eta(t))K(t) \quad . \tag{35}$$

If we take these constraints into account, we thus have three independent variables controlling two independent equations. It would seem apparent that we could easily implement any desired intertemporal allocation of risk bearing, including the optimal one we have just derived. We establish, in fact, a slightly stronger result: we can implement this policy through a financial policy involving constant prices, i.e.,

$$v_t = v_{t+1} = v_{t+\tau} = 1 \quad \text{(without loss of generality)} \quad . \tag{36}$$

To see this, and to help develop our intuition, we begin with the case where there is no risk. Then, (36) together with the equilibrium requirement of equality of returns, implies that we set

$$i(t) = \eta(t) \qquad \text{all } t \tag{37}$$

In this situation, individuals are indifferent to holding bonds or capital in their portfolio.

To induce any generation to consume the correct amount, we increase or decrease T_1 . So long as the marginal propensity to consume is not zero, this will lead to a change in C_1 . Next, we increase or decrease B so that the desired amount of capital accumulation occurs. (Since the two are perfect substitutes, an increase in B induces a dollar-for-dollar decrease in K .) If C_1 and K are set at their correct levels, C_2 must be at its desired level (by (35)). Similarly, any deficit is financed by lump sum taxes on the aged and any surplus is distributed to the aged.

Formally, we find the optimal policy by solving the set of equations.[1]

(38a) $$C_1^*(S(t)) = C_{1t}(w(t) + T_1(t), \eta, T_2(t+1))$$

(38b) $$K^*(S(t)) = w(t) - C_1^*(S(t)) - B(t) - T_1(t)$$

(38c) $$T_1(t) + T_2(t) + B(t) - B(t-1) = G + i(t-1)B(t-1) \quad .$$

Essentially the same argument holds if η is random; now, however, individuals are not indifferent as to the form in which they hold their assets. Changing T_1 alters the level of consumption and savings the first period. Now, however, the fraction of this savings that they wish to hold in the form of capital is not indeterminate. To induce individuals to hold more capital, we have to make the return on capital more attractive relative to

1. (38b) can easily be solved for the optimal sequence of $B(t) + T_1 \equiv z^*(t)$. Then, using (38c), we can rewrite (38a)

$$C_1^*(S(t)) = C_{1t}(w(t) + T_1, \eta; z^*(t+1) + (1+\eta)(z^*(t) - T_1(t)) + G)$$

which we can solve for $T_1^*(t)$, and hence for $B^*(t)$.

money. We can do this by lowering i. By this means we can ensure that C_1 and K_t are "correct" for each S. But this (through the national income identities) assures us that $C_2(t-1)$ is also correct.

We have thus established Proposition V. The optimal intertemporal distribution of income can be implemented by means of a financial policy with a constant price level, provided there is a sufficient flexibility in the imposition of lump sum taxes/subsidies on the young and the old.

VII. The Role of Additional Financial Instruments

Since we have shown that we can obtain the optimal intertemporal distribution of income with a single financial asset, is there any role to be played by the introduction of additional financial assets, e.g., government bonds of differing maturities? We show here that if there is complete flexibility in the imposition of lump sum taxes and subsidies, such an additional financial asset has no effect, but if there are restrictions, say, on the variability of lump sum payments to the aged, then an additional financial instrument can be used to achieve the optimal intertemporal distribution of income.

For simplicity, let our second financial asset be a long-term bond, a perpetuity, paying \$1 every period. The price, q, of these bonds is, however, random, so that the net yield r_t is a random variable. Government policy again entails a rule for the increase or decrease in the supply of these financial instruments, as a function of the state (and possibly history) of the economy.

It is obvious that, in the case where η is not random, and v_t is constant, such a financial instrument is completely redundant. For since i

is constant, the price, q, of this security is fixed, and it is no different from a short-term bond.

In the case where η is random, however, such a security is different from a short-term bond. To show that it is still redundant, we need to rewrite the government's budget constraint.

$$T_1 + T_2 + q(t)(D(t) - D(t-1)) + v(t)(B(t) - B(t-1)) \tag{39}$$

$$= G + i(t-1)B(t-1) + D(t-1)$$

where $D(t)$ is the number of long-term bonds outstanding at date t.

Thus, assume that the government were to fix T_2 at zero (or at any other arbitrary fixed level). Assume the government set T_1 at its previous level, and set

$$\hat{\hat{v}}\hat{\hat{B}} + \hat{\hat{q}}\hat{\hat{D}} = \hat{v}\hat{B} \quad \text{for all dates and states} \tag{40}$$

i.e., made the total value of outstanding government securities the same. We can easily verify that, taking the government budget constraint into account, the value of second period consumption is

$$K(t+1)(1+\eta(t+1)) + T_1 - G + q(t+1)D(t+1) + v(t+1)B(t+1) \quad , \tag{41}$$

which is identical to what it would have been had the government had a single financial asset. (Compare (41) and (25), using (40).)

Thus, the individual's first order condition for savings (first period consumption) is still satisfied (equation (24)) at the original value of C_1 ; and since savings are unchanged, and the value of financial assets is unchanged, capital accumulation is unchanged. By the national income identity

(35), C_2 must be unchanged.[1,2]

The analysis so far has assumed that there is a single type of individual. Is the second financial instrument redundant if individuals differ?

First best optimality would necessitate the government imposing a different set of lump sum taxes/subsidies on each type of individual. We assume that that is not feasible. The addition of a second risky asset may have two effects: it may affect the ability of the economy to efficiently share risks within a generation; and it may affect the intra- and intergenerational distribution of income. In order to abstract from the first effect, let us assume that there is initially a complete set of intragenerational Arrow-Debreu securities markets, so that the marginal rate of substitution between consumption in two different states is the same for all individuals alive contemporaneously. Still, the addition of a second

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1. We assume $\{q(t), i(t), v(t)\}$ adjust to whatever they have to in order for portfolio equilibrium to be established.

2. More formally, we show that the individual's opportunity set is unchanged. For simplicity, we normalize by letting $\hat{\hat{v}}(t) \equiv 1 = \hat{v}(t)$. If the individual sets

$$\hat{B}(t) = \hat{\hat{B}}(t) + \hat{\hat{q}}(t)\hat{\hat{D}}(t)$$

the government's budget constraint is satisfied with unchanged taxes if

$$\begin{aligned} \hat{i}(t)\hat{B}(t) + \hat{B}(t) &= (1+\hat{\hat{i}})\hat{\hat{B}}(t) + \hat{\hat{D}}(t) + \hat{\hat{q}}(t)\hat{\hat{D}}(t) \\ &+ \hat{B}(t-1) - \hat{\hat{B}}(t-1) - \hat{\hat{q}}(t)\hat{\hat{D}}(t-1) \\ &= (1+\hat{\hat{i}})\hat{\hat{B}}(t) + \hat{\hat{D}}(t) + \hat{\hat{q}}(t)\hat{\hat{D}}(t) \quad . \end{aligned}$$

Clearly, any sequence of $\{\hat{C}_1, \hat{C}_2, \hat{K}\}$ which was initially feasible is still feasible (and conversely). Hence, the same $\{C_1, C_2, K\}$ will be chosen: nothing real has changed. This establishes that corresponding to any equilibrium with long-term bonds, there is a corresponding and essentially identical equilibrium without long-term bonds.

financial asset will, in general, have real effects. For assume not. Then it must be the case that the Arrow-Debreu prices are unaffected by the changes in financial policy; but the change in financial policy does result in a change in T_2, and hence in the value of the individuals' endowments. But this change in the value of endowment, at fixed Arrow-Debreu prices, will have real effects, both on C_1 and C_2.

In general, then, adding an additional financial asset will have real effects.

The basic intuition behind this result is that policies which affect aggregate consumption, say, in the second period, in the same way, may have different effects on different individuals. Paying an effectively higher return on long-term bonds, but lowering social security payments, may, on average, have no effect on the consumption of the aged. But the old who are less risk averse and buy risky, long-term bonds are better off under such a policy, and those who do not speculate, and rely on their social security payments, are relatively worse off. But these intragenerational distributional changes have, in turn, real effects on the economy.

Note that for any particular specification of the financial policy of the government, we can calculate the term structure of interest rates, the relationship between the expected return on the long-term bond and the return on the short-term bond; though the normal presumption is that, since the long-term bonds are riskier, they have to yield an expected return which exceeds that on the short-term bond, since, in equilibrium, the yield on the long-term bonds is related to $S(t+1)$, as is $T_2(t)$, it is conceivable that just the opposite result obtains.

It is easy for the government to create additional financial instruments. Assume that the government announces a long-term bond, which when the state is $S(t)$, will yield (in the following period) a return $i(S)$; such a perpetuity will (with the appropriate financial policy) yield a variable return, which will not, in general, be a linear function of the return on short-term bonds and long-term perpetuities with fixed payments. And so long as such instruments represent real additions to the set of assets, and so long as there are fewer such instruments than there are types of individuals in the economy, then these additional instruments will not be redundant.

The basic insight behind the results of this section can be put fairly simply. When all individuals are identical, to "control" the economy, all one needs to do is to control $C_1(t)$ and $K(t+1)$. This requires two instruments, and the availability of age specific lump sum payments and short-term bonds provides us with all the instruments we need. But when individuals are not identical, and we cannot vary the age specific lump sum payments from individual to individual, we have more "objects" we wish to control than we have instruments; i.e., we would like to control $C_1^j(t)$, $C_2^j(t)$, and $K(t)$. Increasing the set of financial instruments, then, does in general increase the real opportunity set of the economy.

The results of this section are summarized in Proposition VI. With a single type of individual, and full flexibility in the imposition of lump sum taxes and subsidies, additional financial instruments (such as long-term bonds) are redundant. When there are more types of individuals, or when there are restrictions on the flexibility of lump sum taxes and subsidies, additional financial instruments are not redundant. The maturity structure of the government debt then has real effects.

Extensions and Concluding Remarks

The object of this paper has been to develop a framework within which alternative financial policies of the government may be analyzed. It is our contention that any meaningful analysis of public financial policy requires the integration of all the important aspects of debt, tax, and social security policy within a single framework, and in particular, requires an intertemporal, stochastic model. There are some circumstances in which we have shown that changes in public financial policy (such as changes in corporate financial policy) have neither real nor financial effects (i.e., all prices remain unchanged). An increase in the supply of bonds gives rise to an exactly equal increase in the demand for bonds. There are other circumstances in which changes in public financial policy have effects on prices, but no real effects. Thus public financial policy affects the rate of inflation, but the rate of inflation has no real consequences. In general, however, public financial policy has real consequences for the intertemporal distribution of risk bearing, and thus for the intertemporal distribution of welfare. Even restricting the government to financial policies with the same expected rate of inflation, and the same expected return to financial assets, changes in financial policy have important real effects on the economy. Indeed, we showed how an appropriately designed public financial policy could be used to implement the first best intertemporal allocation of resources. To do this required, however, complete flexibility in the imposition of lump sum taxes and subsidies on the young and the old. When, for instance, social security payments were not allowed to vary from year to year and from state to state, the first best intertemporal allocation of resources could only be implemented through public financial policy if there was an additional financial instrument.

It should be noticed that in the model we have constructed all individuals have fully rational expectations concerning the nature of future government policies. Yet, in general, in spite of the rational expectations, public financial policy does have real effects. Those models which have concluded that with rational expectations government financial policy is irrelevant reach their conclusions not because of their assumptions concerning how expectations are formed, but rather from the specific structural assumptions of their models. One such assumption which has been extensively criticized is their assumption of complete price flexibility (see Taylor (1980) and Neary-Stiglitz (forthcoming)); results concerning full employment (and hence the inefficacy of monetary policy) are perhaps not surprising in a world with perfect price flexibility, and could be established under a variety of assumptions concerning how expectations are formulated. Here, we have established that even with perfect price flexibility, changes in public financial policy will in general have real effects.

The analysis of this paper raises several further questions of interest. First, we have assumed, throughout, that all taxes are lump sum. In practice, most taxes are distortionary. With lump sum taxes, the intertemporal pattern of the imposition of taxes (on a single individual) makes no difference. When taxes are distortionary, it does. In the absence of uncertainty, for instance, with suitable symmetry and separability assumptions, it would be optimal to levy wage taxes at a constant rate throughout the individual's lifetime. This provides, then, a simple theory of the optimal size of the government debt: government debt simply serves as a "buffer stock" between the optimal pattern of government expenditure and the optimal pattern of tax revenues. The analysis of optimal taxation in the presence of uncertainty is a far more complicated question, which we hope to pursue elsewhere.

Second, it is of interest to know the consequences of alternative simple financial rules. If the government must choose, say, between a rule which maintains prices fixed, and a rule which keeps the real value of the debt fixed, which is preferable?

Third, although we have provided a general result characterizing the optimal pattern of the intertemporal distribution of resources under uncertainty, we have not provided many insights into its detailed structure; this will be required if we are to analyze the structure of optimal public financial policies.

Fourth, although we have discussed the role of additional financial instruments, there are two such securities that merit more detailed attention. We have ignored throughout the role of government debt in facilitating transactions; in particular, we have ignored the distinction between non-interest bearing short-term debt and interest bearing short-term debt. If we introduce money, and assume that it has transactions advantages over interest bearing short-term debt, how are our results affected? The results reported in Stiglitz (1982) suggest that, if anything, introducing debt strengthens the presumption that public financial policy does matter.

A second financial instrument which has received extensive attention in recent years is a government bond with a guaranteed real rate of return. Would the introduction of these indexed bonds make a difference? The analysis of this paper (confirmed by the results in Stiglitz (1982)) suggests that if individuals are essentially identical, then this additional financial instrument is redundant; but that if they differ enough, then providing this extra instrument does expand the real opportunity set of the economy.

Finally, and perhaps most importantly, the analysis of this paper has been conducted within a neoclassical framework, in which there is full employment every period. One of the central issues with which public financial policy has traditionally been concerned is the extent to which it can affect the level of employment and output. To address these questions requires the formulation of a macro-economic model with unemployment. It is likely that at least some of the mechanisms by which public financial policy affects the economy in such circumstances are quite different from those portrayed here.

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